Warsaw University of Technology

Faculty of Power and Aeronautical Engineering

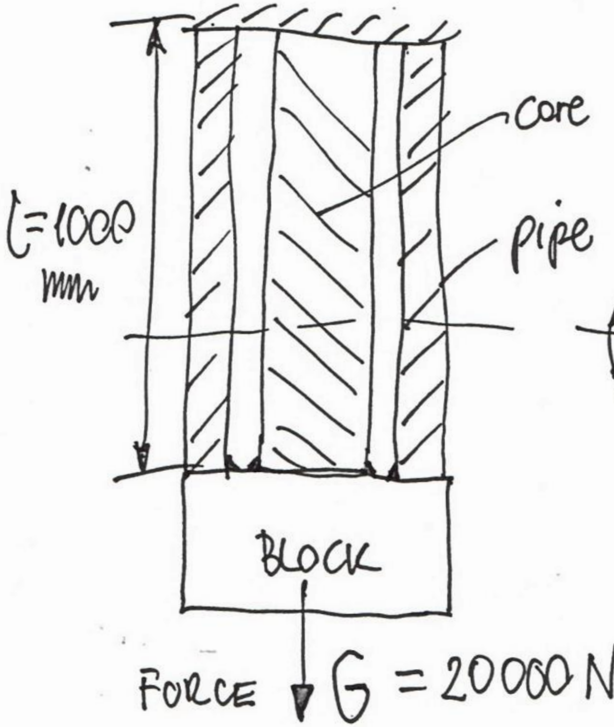
WARSAW UNIVERSITY OF TECHNOLOGY

Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM 2)

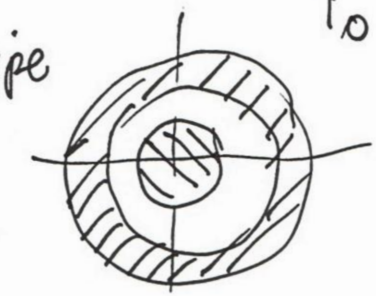
Structural analysis with thermal effect – bar example

EXAMPLE: BUILD A FE MODEL OF A STATICALLY INDETERMINATE BAR STRUCTURE. FIND THERMAL LOAD, STRAINS, STRESSES AND REACTIONS IN THE CORE AND PIPE.



$T = 200°C$

$T_0 = 20°C$



$\Delta T = 200°C - 20°C =$
$= 180°C$

UNIFORM TEMPERATURE DISTRIBUTION

pipe: (steel)

$E_p = 2 \cdot 10^5$ MPa

$\alpha_p = 1.2 \cdot 10^{-5} \frac{1}{°C}$

$A_p = 200\ mm^2$

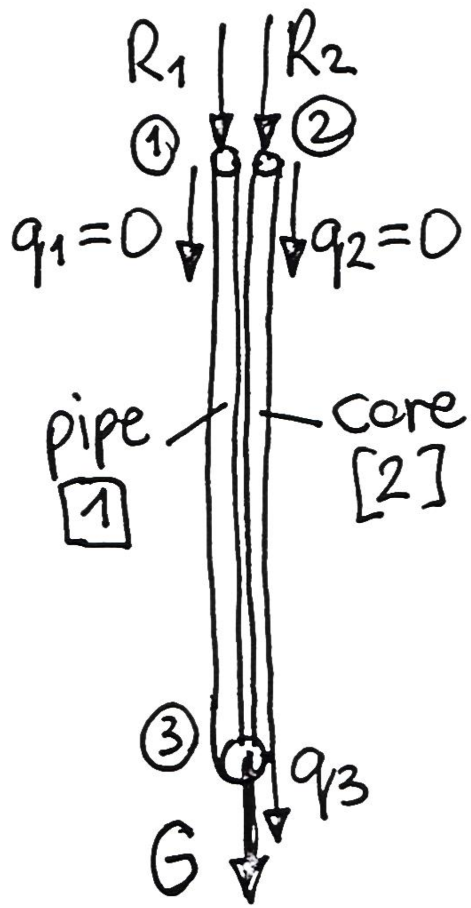
core: (copper)

$E_c = 1{,}06 \cdot 10^5$ MPa

$\alpha_c = 1.58 \cdot 10^{-5} \frac{1}{°C}$

$A_c = 50\ mm^2$

FE MODEL



$$\underset{3\times 1}{\{q\}} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \quad \text{(mm)}$$

$$\{F_s\} = \begin{Bmatrix} R_1 \\ R_2 \\ G \end{Bmatrix}$$

THERMAL LOAD

$\boxed{1}$

$\Delta T_1 = \Delta T$

$\Delta T_3 = \Delta T$

$$\underset{2\times 1}{\{F_T\}_1} = \frac{\Delta T_1 + \Delta T_3}{2} \alpha_p E_p A_p \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \Delta T \alpha_p E_p A_p \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$\boxed{2}$

$\Delta T_2 = \Delta T$

$\Delta T_3 = \Delta T$

$$\{F_T\}_2 = \frac{\Delta T_2 + \Delta T_3}{2} \alpha_c E_c A_c \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \Delta T \alpha_c E_c A_c \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$\underset{3\times 1}{\{F_T\}_1^*} = \Delta T \alpha_p E_p A_p \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix}$$

$$\underset{3\times 1}{\{F_T\}_2^*} = \Delta T \alpha_c E_c A_c \begin{Bmatrix} 0 \\ -1 \\ 1 \end{Bmatrix}$$

$$\underset{3\times 1}{\{F_T\}} = \{F_T\}_1^* + \{F_T\}_2^* = \Delta T \cdot \begin{Bmatrix} -\alpha_p E_p A_p \\ -\alpha_c E_c A_c \\ \alpha_c E_c A_c + \alpha_p E_p A_p \end{Bmatrix}$$

$$[K]_e = \frac{EA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k]_1 = \frac{E_p A_p}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad , \quad [k]_2 = \frac{E_c A_c}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K] = \frac{1}{l} \cdot \begin{bmatrix} E_p A_p & 0 & -E_p A_p \\ 0 & E_c A_c & -E_c A_c \\ -E_p A_p & -E_c A_c & E_p A_p + E_c A_c \end{bmatrix}$$

Boundary conditions:

$$q_1 = 0 \;,\; q_2 = 0$$

$$\underset{3\times3}{[K]} \cdot \underset{3\times1}{\{q\}} = \{F_S\} + \{F_T\}$$

$$\left(\frac{E_pA_p + E_cA_c}{L}\right) \cdot q_3 = G + \Delta T \cdot (\alpha_c E_c A_c + \alpha_p E_p A_p)$$

$$q_3 = \frac{G + \Delta T(\alpha_c E_c A_c + \alpha_p E_p A_p)}{\frac{E_pA_p + E_cA_c}{L}} = 2.682\,mm$$

TOTAL STRAIN :

1st element : $\varepsilon_1 = \dfrac{q_3 - q_1}{l_1} = \dfrac{q_3}{L} = 2.682 \cdot 10^{-3}$

2nd element : $\varepsilon_2 = \dfrac{q_3 - q_2}{l_2} = \dfrac{q_3}{L} \Rightarrow \varepsilon_1 = \varepsilon_2 = \varepsilon$

THERMAL STRAIN:

$\boxed{1}$ $\varepsilon_{T_1} = \alpha_p \cdot \Delta T = 2.16 \cdot 10^{-3}$

$\boxed{2}$ $\varepsilon_{T_2} = \alpha_c \cdot \Delta T = 2.844 \cdot 10^{-3}$

ELASTIC STRAIN:

$\boxed{1}$ $\varepsilon_{e_1} = \varepsilon - \varepsilon_{T1} = 0.522 \cdot 10^{-3}$ (POSITIVE)

$\boxed{2}$ $\varepsilon_{e_2} = \varepsilon - \varepsilon_{T2} = -0.1625 \cdot 10^{-3}$ (NEGATIVE)

STRESS :

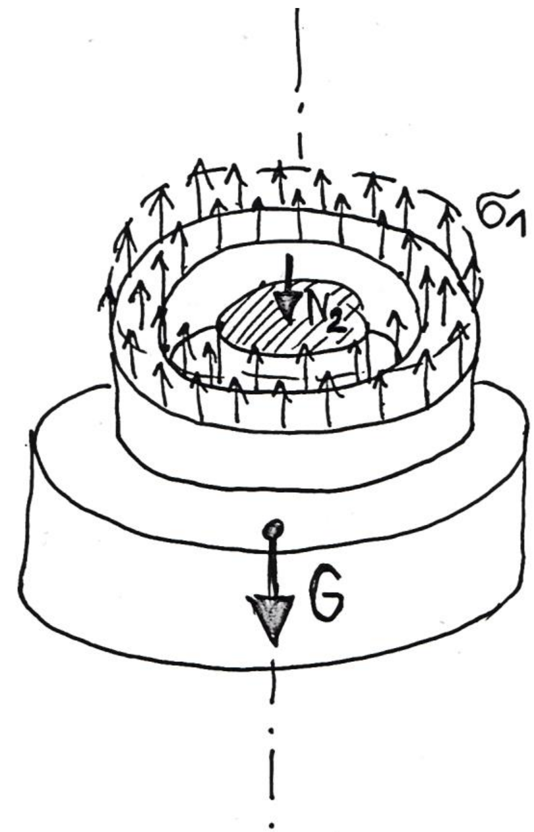
$\boxed{1}$ $\sigma_1 = E_p \cdot \varepsilon_{e_1} = 104.31 \text{ MPa}$

$\boxed{2}$ $\sigma_2 = E_c \cdot \varepsilon_{e_2} = -17.22 \text{ MPa}$

INTERNAL FORCES

$\boxed{1}$ $N_1 = \sigma_1 \cdot A_p = 20861.4 \text{ N}$

$\boxed{2}$ $N_2 = \sigma_2 \cdot A_c = -861.1 \text{ N}$



REACTIONS

$$\underset{3\times 3}{\left[K\right]}\cdot\underset{3\times 1}{\{q\}}=\underset{3\times 1}{\{F\}}$$

I) $$\frac{E_p A_p}{l}\cdot q_1 + 0\cdot q_2 + \left(-\frac{E_p\cdot A_p}{l}\right)q_3 = R_1 - \alpha_p\cdot E_p A_p \Delta T$$

II) $$0\cdot q_1 + \frac{E_c A_c}{l}\cdot q_2 - \frac{E_c A_c}{l} q_3 = R_2 - \alpha_c\cdot E_c A_c \Delta T$$

from I):

$$R_1 = \alpha_p E_p A_p \Delta T - \frac{E_p A_p}{l}\cdot q_3 = -20861.1\,N$$

from II):

$$R_2 = \alpha_c E_c A_c \Delta T - \frac{E_c A_c}{l} q_3 = 861.1\,N$$